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Principles of Wideband Unidirectional Piezoelectric Transducers

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LINCOLN LABORATORY

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ABSTRACT

By interweaving the fingers of an interdigital transducer with an earthed meander line and supplying different voltages, in different phases, to the fingers, a variety of interesting performances may be achieved. Two unidirectional transducers and a diplexing transducer are described as examples of the technique. In order to achieve the performances that theory indicates, a number of technical problems need to be overcome. These problems are pointed out and approaches to their solution are suggested.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

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LIST OF SYMBOLS

ℓ	Distance between centers of successive fingers, or between centers of successive arms of meander line. See Figs. 2 and 5.
x	Distance, measured from the center of the arm of the meander line to the left of the r^{th} strip. x varies from 0 to ℓ.
z	Distance in the propagation direction, measured from the center of the arm of the meander line lying on the left of the zeroth strip.
V_r	RF voltage applied to the r^{th} strip ($0 \leq r \leq n$).
α	Phase difference between the r^{th} and $(r + 1)^{th}$ strips.
n	One less than the total number of strips or, in the case of Fig. 6, strip sites (whether occupied by a strip or not).
B_1, B_2	Excitation coefficients.
β	Phase constant of surface waves in the substrate, propagating in the z direction.
A(z)	Amplitude of surface waves in the substrate.
D	Directivity of a transducer – the ratio of the power radiated in the desired direction to that radiated in the opposite direction.

PRINCIPLES OF WIDEBAND UNIDIRECTIONAL PIEZOELECTRIC TRANSDUCERS

I. INTRODUCTION

Rayleigh waves are commonly launched in piezoelectric materials by means of interdigital electrode structures, as illustrated in Fig. 1. The two large rectangular pads are used as terminals for connection to an external electrical exciting circuit. One pad is earthed, the other being excited by an RF voltage.

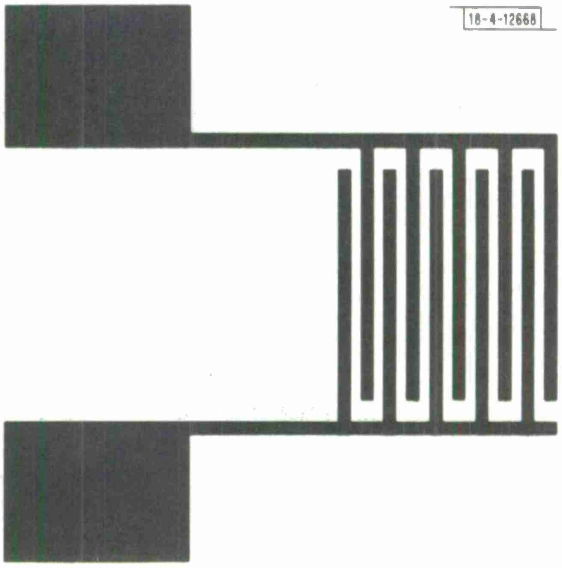

Fig. 1. Conventional interdigital surface-wave transducer, with ten fingers.

The coupling efficiency of a transducer of this type depends on the number of fingers, on their length, and on the ratio of the finger width to the finger spacing. Some studies of this question have been made by Collins, Shaw, and Smith* and by Coquin and Tiersten,† but much more needs to be done. The number of fingers should be fairly large for a reasonable coupling efficiency, but as the number of fingers becomes large the bandwidth becomes small. Also, the transducer radiates equally in either direction, so that half the power is lost. A transducer which radiated in only one direction would effect a considerable saving in power; this would increase the number of transducing operations which could be performed in a system before amplification became necessary, and would reduce or eliminate the problem of disposing of the energy radiated in the wrong direction.

A unidirectional transducer described by Collins, Shaw, and Smith* consists of two simple conventional interdigital transducers arranged so as to generate waves traveling over the same path. By separating the transducers by a certain amount and exciting them in the appropriate

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*J. H. Collins, H. J. Shaw, and W. R. Smith, "Scattering and Transfer Matrix Analysis for Acoustic Surface Wave Transducers," in "Wideband Acoustic Surface Wave Couplers at Microwave Frequencies," M. Chodorow, Microwave Laboratory, W. W. Hansen Laboratories of Physics, Report ML-1736, Stanford University, Stanford, California (April 1969).

†G. A. Coquin and H. F. Tiersten, "Analysis of the Excitation and Detection of Piezoelectric Surface Waves in Quartz by Means of Surface Electrodes," J. Acoust. Soc. Am. 41, 921-939 (1964).

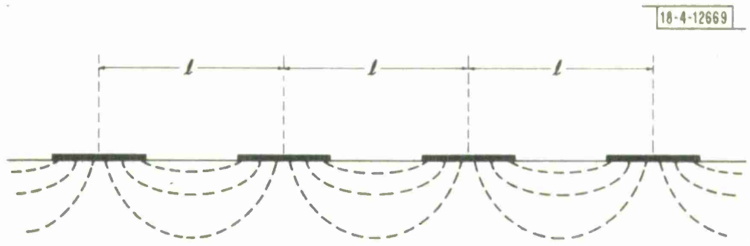

Fig. 2. Section through a transducer perpendicular to the lengths of the fingers, showing the electric field configuration in the substrate.

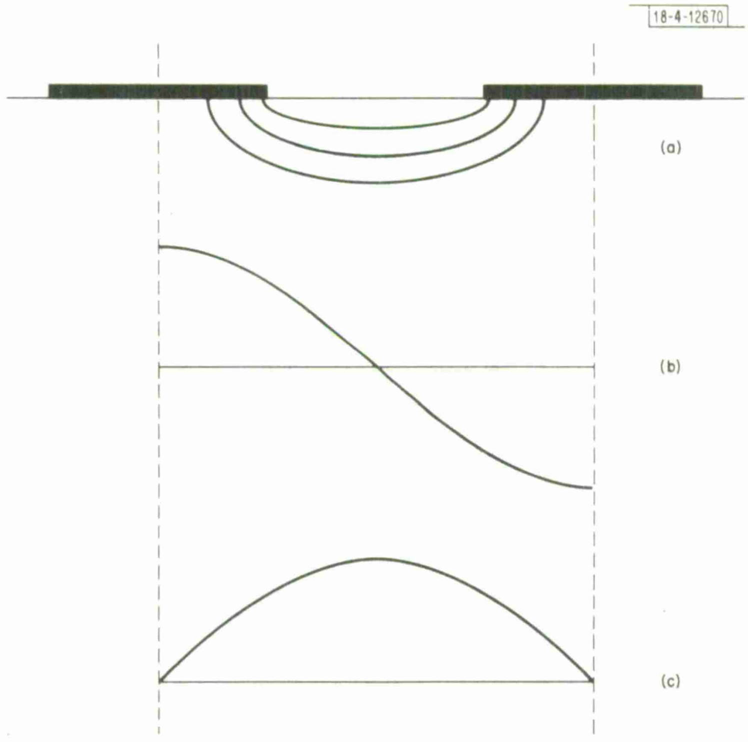

Fig. 3. Resolution of the field between two successive electrodes into components normal and parallel to the substrate surface. (a) Field lines; (b) normal field component; (c) parallel field component.

relative phases, the wave-trains generated in the piezoelectric material can be made to interfere constructively in one direction and destructively in the other. Collins, Shaw, and Smith calculated the directivity (defined as power in the wanted direction divided by power in the unwanted direction) to vary between 8 and 11 dB over a bandwidth of about 7%.

Conventional interdigital transducers and the directional transducer of Collins, Shaw, and Smith operate on very simple principles. Alternate fingers are earthed, and the remaining fingers are excited at the same RF potential. This leaves a number of degrees of freedom unexploited. It is to be expected that if the parameters of the transducer are varied a great variety of filtering performances might be realized. Parameters which might be varied are the finger spacing, which in existing transducers is uniform but need not be so; the ratio of finger width to finger separation; and the amplitudes and phases of the voltages applied to the electrodes. In the present report, some calculations will be made to show the directivities of unidirectional transducers that may be achieved by exciting the fingers of a transducer at different voltages. How to supply these voltages is another question. Although it is discussed in part below, some problems remain to be solved, although they do not appear to be insuperable.

II. EXCITATION OF THE TRANSDUCER

We assume that the fingers, of metal, are thick enough to act as perfect conductors of electricity, but so thin that they do not load the substrate mechanically. A section through the transducer, transverse to the lengths of the fingers, is shown in Fig. 2. If different voltages are applied to the fingers, electric field lines will be established in the substrate with the general form shown by the dashed curves. These fields can be resolved into a vertical and a horizontal component, and we shall assume that the vertical component, between any two successive fingers, is proportional to $\cos(\pi x/\ell)$ while the horizontal component is proportional to $\sin(\pi x/\ell)$ (see Fig. 3). This amounts to taking only the first term of the Fourier spectrum of each field component. This may seem a rather drastic limitation, but we shall justify it in Section VII in the light of the results obtained.

We have been speaking of voltages on electrodes, but actually it is fields within the piezoelectric material which generate elastic waves. This is not the trivial point it may appear at first sight. Mathematically there is no difficulty in calculating the voltages that must be applied to the individual fingers in an array in order to obtain a desired electric field distribution. The difficulty is the practical one of achieving those voltages. For the exciting voltages we are going to need, the amplitudes and phases of the voltages that would be required on a conventional interdigital structure turn out to be quite complicated. This would necessitate complicated excitation circuits, and to obtain the phase relationship resistors would have to be used in conjunction with reactors; this would pose a problem with thermal dissipation.

The realization of a design could be made much simpler if the exciting field could be made proportional to the voltage applied to an electrode. This can be done by interweaving the fingers of the transducer with a meander line maintained at earth potential (Fig. 4). The field lines then go both ways from a live strip, ending on the two arms of the meander line which enclose it. The field lines for the r^{th} strip are as shown in Fig. 5; the two electric field components at any point x are in the same phase as the applied voltage, and proportional to it in magnitude.

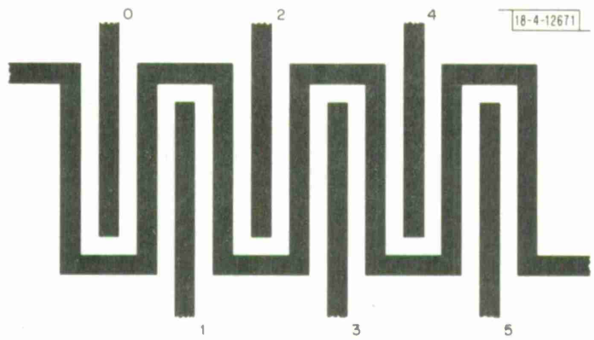

Fig. 4. Interdigital transducer with interwoven meander line.

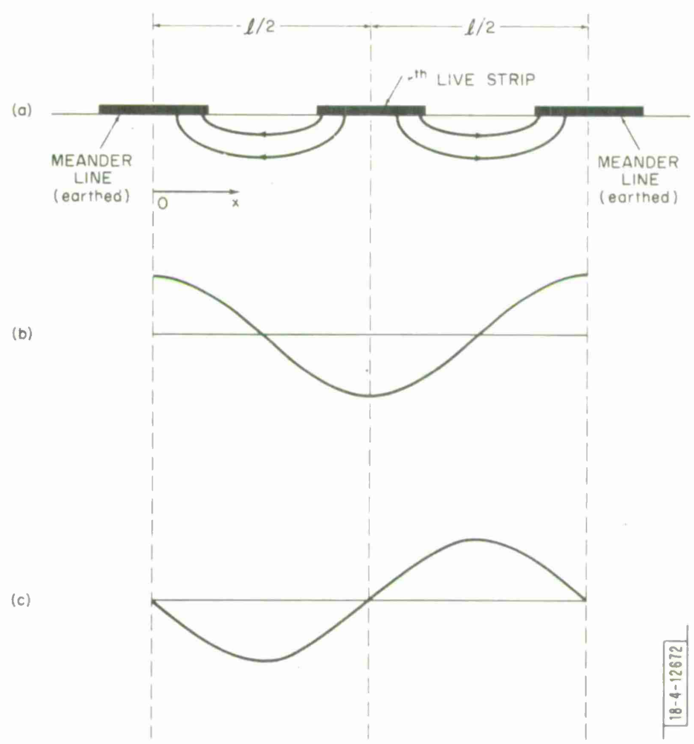

Fig. 5. Field configuration around the r^{th} live strip. (a) Field lines for r^{th} live strip; (b) vertical component of electric field; (c) horizontal component of electric field.

III. RESPONSE OF THE TRANSDUCER

We now require to calculate the amplitude, at a point distant z from the start of the transducer, of a wave excited by the transducer. The start of the transducer, i.e. the point $z = 0$, is the arm of the meander to the left of the strip 0 in Fig. 4.

The amplitude at z due to an element dx at x in the region of the r^{th} strip is proportional to

$$V_r e^{j(\omega t + r\alpha)} e^{\pm j\beta(z - r\ell - x)} \left(B_1 \sin \frac{2\pi x}{\ell} + B_2 \cos \frac{2\pi x}{\ell}\right) dx \tag{1}$$

where $0 < x < \ell$. $V_r e^{j(\omega t + r\alpha)}$ is the voltage applied to the r^{th} finger ($r = 0, 1, 2, \ldots n$), a phase shift α occurring between each finger and the next. We are taking α as the same for all r; this is convenient mathematically, and is probably the simplest condition to realize practically. However, there is no reason in principle why α may not be different for different values of r, giving the designer an extra degree of freedom. The factor $e^{\pm j\beta(z - r\ell - x)}$ is the phase change due to propagation from the point $r\ell + x$ to the point z, either in the positive direction (minus sign) or in the negative direction (plus sign). The trigonometric factors arise from the variation with x of the electric field components. B_1 and B_2 are scale factors representing the efficiencies of excitation; we shall comment on these quantities in Section VII.

The net result $A(z)$ at the point z is obtained by integrating with respect to x and summing over all r. Thus we can write

$$A(z) = \sum_{r=0}^{n} \left\{ e^{j(\omega t + r\alpha)} \, e^{\pm j\beta(z - r\ell)} \, V_r \int_0^{\ell} e^{\mp j\beta x} \left[B_1 \sin \frac{2\pi x}{\ell} + B_2 \cos \frac{2\pi x}{\ell}\right] dx \right\} \tag{2}$$

Integrating,

$$A(z) = \frac{2[B_2\beta \mp 2jB_1\pi/\ell]}{\beta^2 - 4\pi^2/\ell^2} \sin \frac{\beta\ell}{2} \, e^{\mp j\beta\ell/2} \, e^{j(\omega t \pm \beta z)} \sum_{r=0}^{n} V_r \, e^{jr(\alpha \mp \beta\ell)} \tag{3}$$

The quantity of interest is not $A(z)$ itself but the amplitude $|A|$. This is

$$|A| = \left| \frac{2\sqrt{B_2^2\beta^2 + B_1^2 \, 4\pi^2/\ell^2}}{\beta^2 - 4\pi^2/\ell^2} \sin \frac{\beta\ell}{2} \sum_{r=0}^{n} V_r \, e^{jr(\alpha \mp \beta\ell)} \right| \tag{4}$$

Equation (3) is true for complex values of β, which will obtain if the piezoelectric material has an appreciable loss. Complex values of B_1 and B_2 are also permitted in Eq. (3); non-real values would occur if electric fields in different directions in the piezoelectric material excited mechanical waves with different phases, which could be the case when elastic loss was present. In going from Eq. (3) to Eq. (4) it has been assumed that no loss is present, so that β is real and B_1 and B_2 are complex numbers having the same argument (which may be zero, when B_1 and B_2 would both be real).

We are interested to know the absolute value of $|A|$ and the directivity, which is the ratio of $|A|^2$ in the forward direction to $|A|^2$ in the reverse direction. Further progress depends on the assumption of particular values for the V_r and α. Some examples are considered in the following sections.

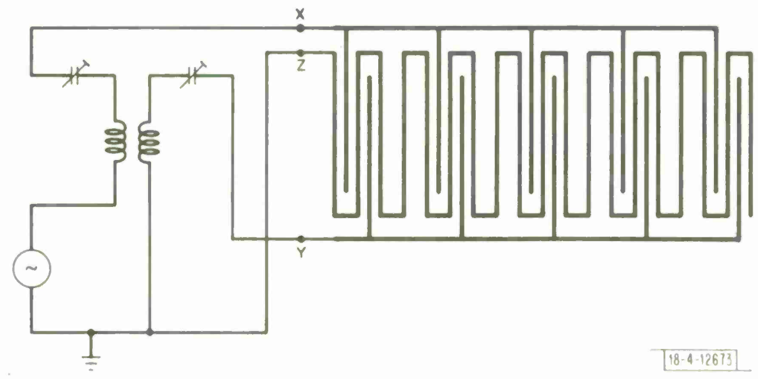

Fig. 6. Unidirectional transducer with input circuit.

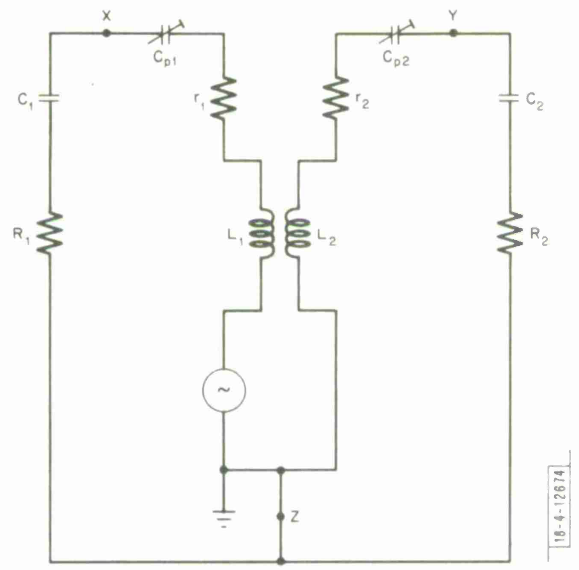

Fig. 7. Equivalent circuit of Fig. 6. The terminals X, Y, and Z correspond to the terminals X, Y, and Z in Fig. 6. C_1 and R_1, C_2 and R_2, are the equivalent circuits of the upper and lower transducer fingers respectively.

IV. AN IMPROVED UNIDIRECTIONAL TRANSDUCER

Figure 6 shows an improvement on the unidirectional transducer of Collins, Shaw, and Smith. The meander is maintained at earth potential, and the upper and lower fingers are excited respectively by the secondary and primary of the transformer. The equivalent circuit is shown in Fig. 7. C_1 and R_1 represent the capacity between the upper fingers and the meander and the radiation resistance of this structure; similarly C_2 and R_2 for the lower fingers. L_1 and L_2 are the effective inductances of the transformer windings, and r_1 and r_2 represent the resistive loss in the transformer. C_{p1} and C_{p2} are padding condensers which, with C_1 and C_2, allow L_1 and L_2 to be tuned.

If $r_1 << R_1$, $r_2 << R_2$, and at the operating frequency the net reactance in the circuit is small compared with R_1 and R_2, the voltages appearing across R_1 and R_2 will be in quadrature, and if the input circuit is properly balanced they will be equal in magnitude. We now have, in effect, two transducers separated by only a quarter-wavelength, instead of by a quarter-wavelength plus the length of one transducer as in the case of the unidirectional transducer of Collins, Shaw, and Smith. Because the transducer is not significantly increased in length, the halving of the bandwidth that occurs with the transducer of Collins, Shaw, and Smith will not occur in the present case. We shall comment further on the electrical circuitry in Section VII.

For the response of this transducer we have

$$\alpha = \pi/2 \tag{5}$$

$$\left.\begin{aligned} V_o = V_1 = V_4 = V_5 = \ldots = V_{4q} = V_{4q+1} = \ldots = V_{2-1} = V_n = V \\ V_2 = V_3 = V_6 = V_7 = \ldots = V_{4q+2} = V_{4q+3} = \ldots = V_{n-3} = V_{n-2} = 0 \end{aligned}\right\} . \tag{6}$$

and then, writing 4p for $n - 1$,

$$\sum_{r=0}^{n} V_r \, e^{jr(\alpha \mp \beta \ell)} = V(1 + je^{\mp j\beta\ell}) \, e^{j2p[(\pi/2) \mp \beta\ell]} \, \frac{\sin[(p+1)(\pi \mp 2\beta\ell)]}{\sin(\pi \mp 2\beta\ell)}$$

so that

$$\left| \sum_{r=0}^{n} V_r \, e^{jr(\alpha \mp \beta\ell)} \right| = \left| V\sqrt{2(1 \pm \sin\beta\ell)} \, \frac{\sin[2(p+1)\beta\ell]}{\sin 2\beta\ell} \right| \tag{7}$$

Substituting this into Eq. (4), the response function is obtained as

$$|A| = \frac{2V\sqrt{2}\sqrt{B_2^2\beta^2 + B_1^2 \, 4\pi^2/\ell^2}}{\beta^2 - 4\pi^2/\ell^2} \sin\frac{\beta\ell}{2} \, \frac{\sin(2p+2)\beta\ell}{\sin 2\beta\ell} \sqrt{1 \pm \sin\beta\ell} \tag{8}$$

where $p = (n - 1)/4$ = one less than the number of fingers on one side of the meander. In the case illustrated in Fig. 6, n is 17 and p is 4, corresponding to five finger-pairs in each of the separate transducers in the Collins-Shaw-Smith unidirectional transducer.

The amplitude in the forward direction is given by taking the plus sign under the square root in Eq. (8). Some results, for various values of B_1 and B_2, are shown in Fig. 8, referred to the value of $|A|$ at $\beta\ell = \pi/2$ as a standard. These results do not take into account any insertion loss due to inefficiency of coupling or to mismatch of the transducer to the electrical input circuit.

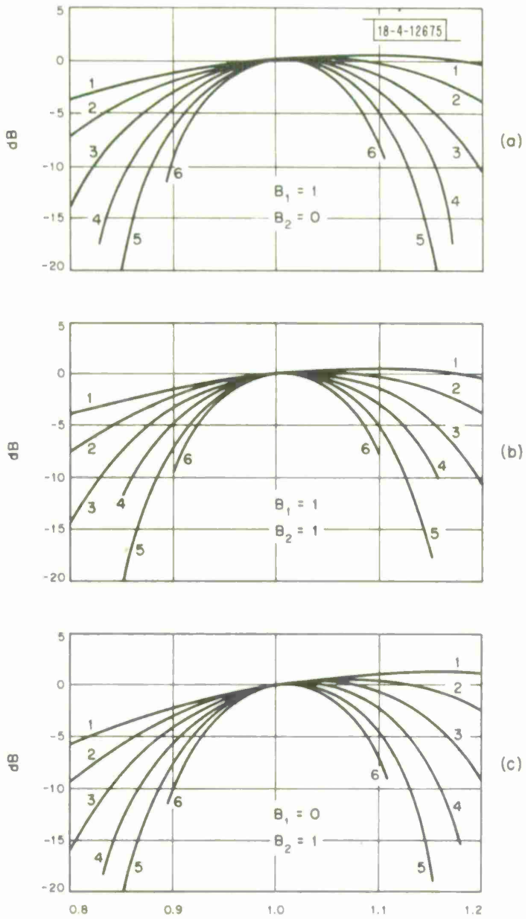

Fig. 8. Amplitude response of the transducer illustrated in Fig. 6. Abscissae $2\beta\ell/\pi$. Parameter $p = (n-1)/4$.

Nor has the effect of the input circuit been taken into account. Corresponding to the 7% bandwidth obtained by Collins, Shaw, and Smith for five finger-pairs, we may expect a bandwidth of about 14%, limited by the response of the input circuit. Over this range, with $p = 4$, it is seen that the amplitude of the transducer response does not vary by more than 3 dB, whatever the relative magnitude of B_1 and B_2.

The directivity D is the ratio of $|A|^2$ in the forward direction to $|A|^2$ in the reverse direction:

$$D = \frac{1 + \sin\beta\ell}{1 - \sin\beta\ell} \tag{9}$$

Clearly D is independent of B_1 and B_2; this means that it is a function only of the electrode spacings and is independent of the nature of the excitation produced by each separate electrode. Notice, too, that D is independent of p or n, i.e. of the length of the transducer in the direction of the propagation, whereas the unidirectional transducer of Collins, Shaw, and Smith has a bandwidth for directivity which decreases with increasing n. Some curves of D against $\beta\ell$ are shown in Fig. 9.

The abscissae in Figs. 8 and 9 are proportional to $\beta\ell$, and $\beta\ell = 2\pi\ell/\lambda = \omega\ell/v_\varphi$, where v_φ is the phase velocity of the surface waves. Since these are non-dispersive, $\beta\ell$ is proportional to frequency, so that the abscissae in Figs. 8 and 9 may be regarded as normalized frequencies.

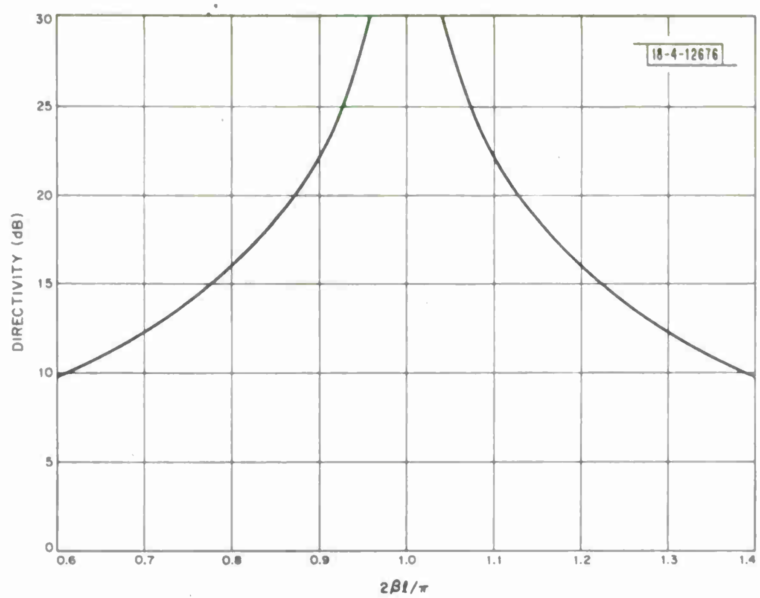

Fig. 9. Directivity of the unidirectional transducer illustrated in Fig. 6. This is independent of p.

Over the bandwidth of 14%, limited by the amplitude response of the combined transducer and electric circuit, Fig. 9. indicates a directivity of over 25 dB. Similar calculations for the Collins-Shaw-Smith transducer give a directivity of 9.6 dB over the 7% bandwidth, which agrees with their theoretical results indicating a directivity varying between 8 and 11 dB over the 7% band.

V. UNIDIRECTIONAL TRANSDUCER WITH BINOMIAL EXCITATION VOLTAGES

The results of Section IV indicate a considerable improvement in the performance of unidirectional transducers by the use of the meander principle. At the end of Section I a number of variables are listed, and of these the only one exploited in the device of Section IV is the phase of the voltages applied to the fingers; even in this case the phase relationships were chosen in a very simple fashion. In the present section we shall show that the performance of the transducer can be still further improved by using excitation voltages which are proportional to the binomial coefficients. The realization of such improvements depends on the associated electric circuitry; this question will be considered later (Section VII). In the present section we confine our attention to the performances that the transducers themselves are capable of, if the required voltages can be delivered to the electrodes.

We consider a transducer of the form shown in Fig. 4, with n + 1 electrodes numbered from 0 to n; the transducer illustrated has n = 4. To the r^{th} electrode we apply a voltage ${}_nC_r \cdot Ve^{j(\omega t+r\alpha)}$, with r = 0, 1, 2, . . . n. Then

$$\sum_{r=0}^{n} V_r\, e^{jr(\alpha \mp \beta\ell)} = V \sum_{r=0}^{n} {}_nC_r\, e^{jr(\alpha \mp \beta\ell)}$$

$$= V[1 + e^{j(\alpha \mp \beta\ell)}]^n$$

$$= V \exp[\tfrac{jn}{2}(\alpha \mp \beta\ell)]\; 2^n \cos^n[\tfrac{1}{2}(\alpha \mp \beta\ell)]$$

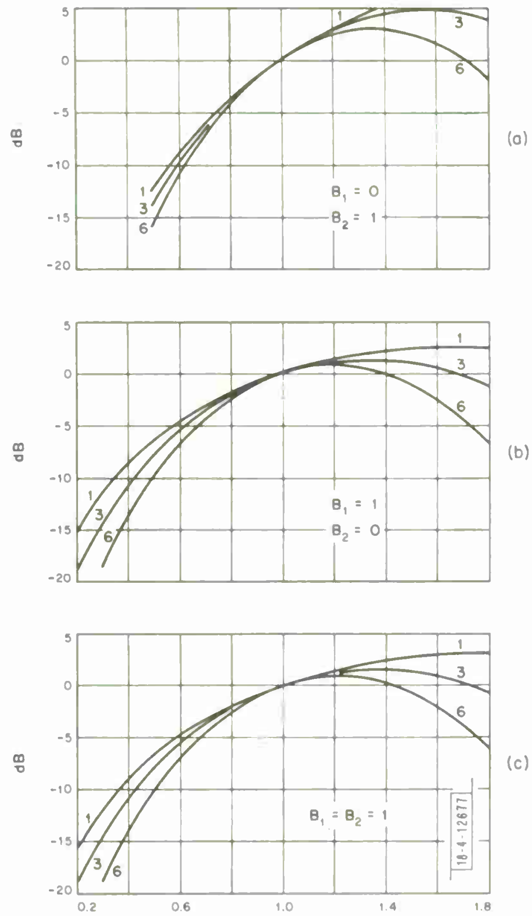

Fig. 10. Amplitude response of binomial unidirectional transducer. Abscissae $2\beta\ell/\pi$. Parameter n, where n + 1 is the number of fingers.

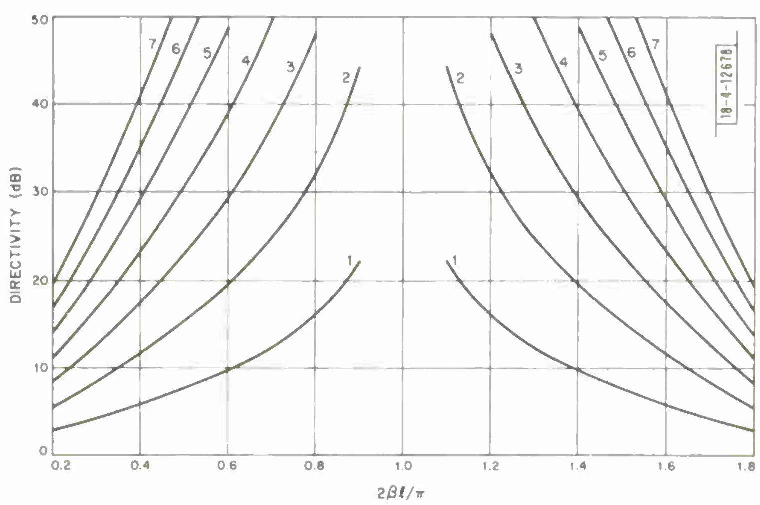

Fig. 11. Directivity of binomial unidirectional transducer. Parameter n, where n + 1 is the number of fingers.

and Eq. (4) becomes

$$|A| = \frac{2^{n+1} V \sqrt{B_2^2 \beta^2 + B_1^2 \, 4\pi^2/\ell^2}}{\beta^2 - 4\pi^2/\ell^2} \left| \sin \frac{\beta\ell}{2} \cos^n [\tfrac{1}{2} (\alpha \mp \beta\ell)] \right| \tag{10}$$

and from this the directivity is readily seen to be

$$D = \left| \frac{\cos^{2n} [\frac{1}{2} (\alpha - \beta\ell)]}{\cos^{2n} [\frac{1}{2} (\alpha + \beta\ell)]} \right| \tag{11}$$

The directivity is a maximum when the deonominator vanishes, so at the mid-band frequency $\frac{1}{2}(\alpha + \beta\ell) = \pi/2$. For any other value of $\beta\ell$, D is maximized if $\alpha = \pi/2$. Equation (10) then becomes, for the forward amplitude,

$$|A| = \frac{2^{n+1} V \sqrt{B_2^2 \beta^2 + B_1^2 \, 4\pi^2/\ell^2}}{\beta^2 - 4\pi^2/\ell^2} \sin \frac{\beta\ell}{2} \cos^n (\frac{\pi}{4} - \frac{\beta\ell}{2}) \tag{12}$$

Equation (11) becomes

$$D = \left| \frac{\cos^{2n} (\frac{\pi}{4} - \frac{\beta\ell}{2})}{\cos^{2n} (\frac{\pi}{4} + \frac{\beta\ell}{2})} \right|$$

which may be written

$$D = \left| \frac{\cos (\beta\ell/2) + \sin (\beta\ell/2)}{\cos (\beta\ell/2) - \sin (\beta\ell/2)} \right|^{2n} \tag{13}$$

Figure 10 shows the amplitude as a function of $\beta\ell$ or normalized frequency, and Fig. 11 shows the directivity. The predicted directivities show a considerable improvement, both in magnitude and bandwidth, over that for the transducer treated in Section IV. The amplitude responses show a considerable variation with frequency. While this appears disappointing at first sight, it must be remembered that no care has been taken in the design over the amplitude response, and that a number of degrees of freedom are available to the designer. An improvement could be obtained, possibly at the cost of a somewhat poorer directivity, by methods similar to those described elsewhere by the author.* However, even without this more sophisticated design it can be seen from Figs. 10 and 11 that for n = 3, over a band from $2\beta\ell/\pi = 0.85$ to 1.55 (58%), the directivity is predicted to be greater than 18 dB, while the amplitude response varies by only 3 dB if $B_1 \geqslant B_2$. This is already a substantial improvement on the transducer of Section IV.

VI. A DIPLEXING TRANSDUCER

With an arrangement similar to that considered in Section V and illustrated in Fig. 4, it is possible to make a diplexing transducer. This is a transducer which is unidirectional in one direction at one frequency and in the opposite direction at another frequency. Realization of the device depends on supplying the correct voltages to the electrodes.

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* R. A. Waldron, "Non-Uniform Waveguides," Chap. 9 in Theory of Guided Electromagnetic Waves (Van Nostrand, in production).

To illustrate the principle, consider the case of four fingers (n = 3), excited with a phase difference of $\pi/2$ between one electrode and the next. Let the exciting voltages be V_o, $V_1 e^{j\pi/2}$, $V_2 e^{2j\pi/2}$, $V_3 e^{3j\pi/2}$. Substituting these values into Eq. (4), the forward amplitude and the directivity are obtained in terms of the three ratios V_1/V_o, V_2/V_o, V_3/V_o, which are to be chosen to give the required performance.

The directivity is

$$D = \left| \frac{V_o + jV_1 e^{-j\beta\ell} - V_2 e^{-2j\beta\ell} - jV_3 e^{-3j\beta\ell}}{V_o + jV_1 e^{j\beta\ell} - V_2 e^{2j\beta\ell} - jV_3 e^{3j\beta\ell}} \right|^2$$

Writing $v_1 = V_1/V_o$, $v_2 = V_2/V_o$, $v_3 = V_3/V_o$, $S = \sin\beta\ell$, this becomes

$$D = \left| \frac{\begin{Bmatrix} (1-v_2)^2 + (v_1 - v_3)^2 \\ + 4S^2(v_2 + v_1 v_3) \end{Bmatrix} + 2S \begin{Bmatrix} (v_1 - 3v_3) + v_2(v_1 + v_3) \\ + 4S^2 v_3 \end{Bmatrix}}{\begin{Bmatrix} (1-v_2)^2 + (v_1 - v_3)^2 \\ + 4S^2(v_2 + v_1 v_3) \end{Bmatrix} - 2S \begin{Bmatrix} (v_1 - 3v_3) + v_2(v_1 + v_3) \\ + 4S^2 v_3 \end{Bmatrix}} \right| \qquad (14)$$

Obviously $D = 1$ when $S = 0$, i.e. when $\beta\ell = \pi$. If $\beta\ell < \pi$, S is positive and $D > 1$. If $\beta\ell > \pi$, S is negative and $D < 1$. Thus at frequencies such that $\beta\ell < \pi$ the transducer will be directive in one sense, and at frequencies such that $\beta\ell > \pi$ it will be directive in the opposite sense. There are three quantities at the designer's disposal, v_1, v_2, and v_3. These can be chosen to satisfy three requirements. One requirement will be the directivity at two values of $\beta\ell$ equally spaced above and below π. S will be of the same magnitude but opposite sign for these, so that one value of D will be the reciprocal of the other and this amounts to only one condition. Two other conditions might be values of forward amplitude or of directivity at other values of $\beta\ell$ – there are many possibilities. Since the numerator and denominator of the directivity are cubics in S, there will in general be three roots, so that in $\pi/2 < \beta\ell < \pi$ there will be three poles of D (infinite forward directivity) and in $\pi < \beta\ell < 3\pi/2$ there will be three zeros of D (infinite reverse directivity). One way in which the conditions might be stated is to specify the values of S, and hence of $\beta\ell$, at which those poles and zeros are to occur.

We shall not pursue these questions here, but to give a suggestion of what might be achieved let us take $V_3 = V_o$, $V_2 = V_1$, so that $v_3 = 1$, $v_2 = v_1 = v$. We now have only one disposable parameter. Let us impose the condition

$$D = \infty \text{ when } S = S_o << 1 \quad . \qquad (15)$$

It follows that $D = 0$ when $S = -S_o$. Equation (14) becomes

$$D = \frac{[(1-v)^2 + 4S^2 v] + S[(v-1)(v+3) + 4S^2]}{[(1-v)^2 + 4S^2 v] - S[(v-1)(v+3) + 4S^2]} \qquad (16)$$

and the denominator of the right-hand side is to vanish when $S = S_o$. This gives a quadratic in v with two equal roots which are

$$v = 1 + 2S_o \tag{17}$$

Substituting this value of v into Eq. (16) we obtain for the directivity

$$D = \left| \frac{(S_o + S)^2(1 + S)}{(S_o - S)^2(1 - S)} \right| \quad . \tag{18}$$

The corresponding amplitude function is

$$|A| = \left| 16 \sin \frac{\beta\ell}{2} \; (S_o \pm S)\sqrt{1 \pm S} \; \frac{\sqrt{B_2^2\beta^2 + B_1^2 \, 4\pi^2/\ell^2}}{\beta^2 - 4\pi^2/\ell^2} \right| \tag{19}$$

where the plus signs are to be taken for $\beta\ell < \pi$ and the minus signs for $\beta\ell > \pi$.

As an example, suppose that the pole and zero of D are to be at frequencies differing by 10%. The values of $\beta\ell$ at these frequencies are, then, 0.95π and 1.05π, and

$$S_o = \sin(0.95\pi) = 0.15643 \quad . \tag{20}$$

Equation (17) then gives

$$v = 1.31286 \quad . \tag{21}$$

Figure 12 shows the directivity as a function of $\beta\ell$ or normalized frequency, and the amplitude of the forward wave is shown in Fig. 13 for various values of B_1 and B_2.

Where the directivity pole and zero occur, at $\beta\ell = 0.95\pi$ and 1.05π, the amplitude curves are steep, and where the amplitude is fairly flat, the directivity is only 10 dB. Thus the device is either narrow-band or not highly directive. In an improved design, one might impose conditions on the minima of the directivity curves or on the positions of the maxima of the amplitude curves.

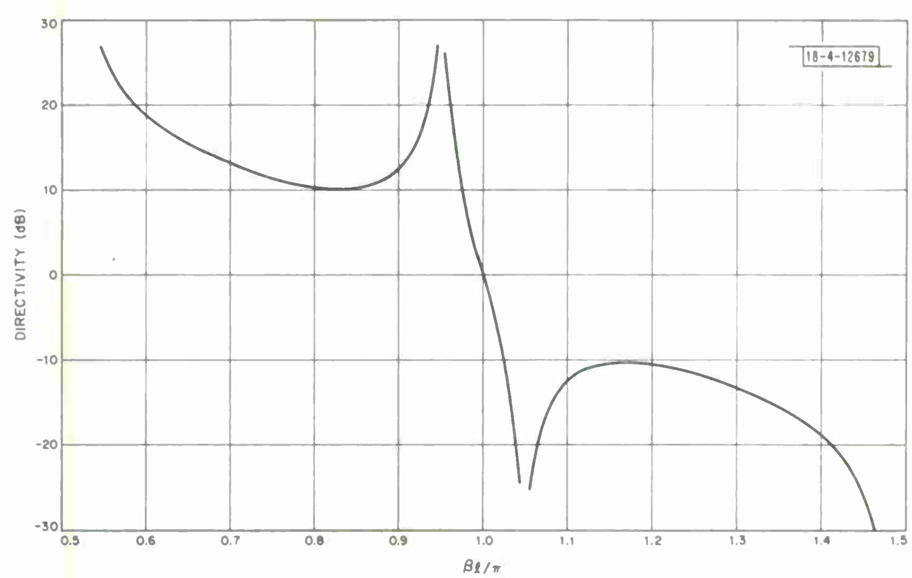

Fig. 12. Directivity of the diplexing transducer.

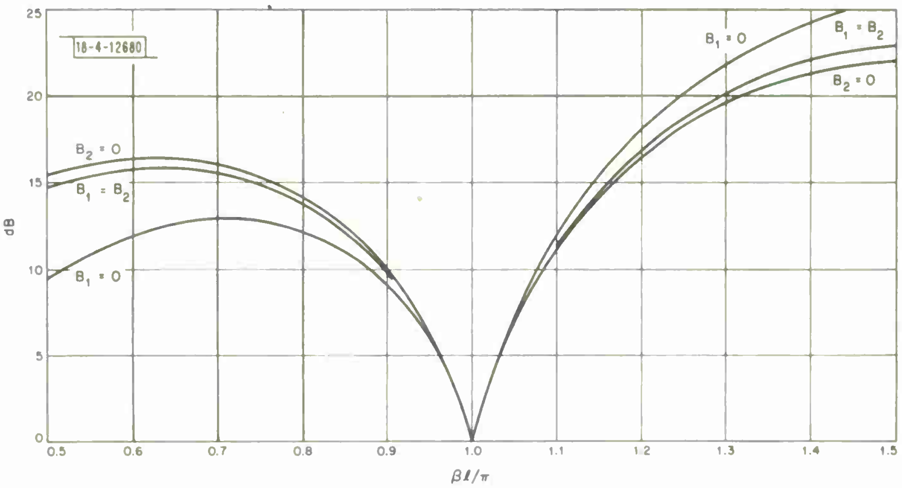

Fig. 13. Amplitude functions of the diplexing transducer.

VII. DISCUSSION

The directivity of a unidirectional transducer of the type considered in this report, i.e. using an interdigital structure interwoven with a meander line, depends only on the geometrical arrangement of the electrodes and on the magnitudes and phases of the voltages applied to them, not on the form of the electric field between electrodes nor on the manner by which the field excites elastic waves in the piezoelectric substrate. This can be seen in the cases treated, and from the theory given in Section III it is clear that the same will be true for any transducer of this type. Thus, from the point of view of directivity, the mechanism of excitation need not be considered.

The amplitude response in the forward direction does depend to a certain extent on the manner of excitation, i.e. on the form of the field lines sketched in Fig. 5 and on the efficiency of coupling from the acoustic wave to the longitudinal and transverse components of electric field. The relative coupling efficiencies determine the relative magnitudes of B_1 and B_2, but Figs. 8, 10, and 13 show that the difference between B_1 and B_2 makes a relatively small contribution to the overall variation of $|A|$ with frequency. Most of the variation in $|A|$ is due not to the manner of excitation by each finger, but to the variation with frequency of the relation between electrode spacing and wavelength. Attention to the electrode spacing (distance between centers) and to the magnitudes and phases of the exciting voltages will improve the amplitude characteristic, but in making adjustments their effects on the directivity characteristic must also be taken into account.

The above considerations justify the assumption made in Section II that only the first terms of the Fourier spectrum of the exciting voltages need to be considered. The exact form of the exciting field, which depends on the configuration of individual electrodes (principally, the ratio of electrode width to gap width), now remains as a parameter which may be adjusted for matching purposes, without greatly altering the transducer response functions.

With all three of the unidirectional transducers discussed in this report it is necessary to supply voltages to different electrodes in phase quadrature, and we have seen how this can be done by means of a transformer in the case of the transducer discussed in Section IV. A transformer can be used in the other two cases (Sections V and VI) too, but since voltages of differing

magnitudes are required, and voltages in anti-phase as well as in quadrature, the transformer will require tappings on both primary and secondary. Such a transformer is illustrated in Fig. 14 for the case of four fingers (n = 3). The switch enables the generator (or load, in the case of an

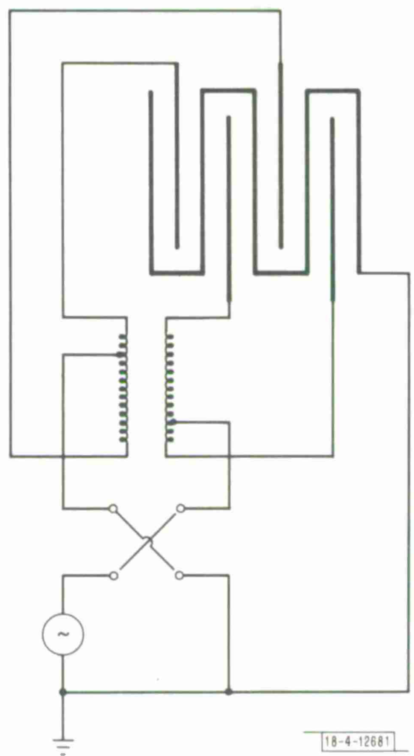

Fig. 14. Exciting circuit for the transducer of Fig. 4.

output transducer) to be connected to either winding of the transformer, so that the phase change may be reversed, thereby reversing the sense of the forward direction of the elastic waves excited by the transducer.

There is now a matching problem, which requires some discussion. Each finger will require its own tap on the transformer. This poses a serious problem if there are a large number of fingers, because the transformer must be small, with only a small number of turns, in order to keep the inductance small which is necessary to obtain the correct phase relations. The performance of the transducer is sensitive to small errors in the voltages applied to the fingers, so that the taps must be very accurately placed. It follows that there must be few taps and correspondingly few electrodes. Moreover, it can be seen from Fig. 10 that the amplitude response is the better, the smaller the number of electrodes.

A small number of electrodes, however, means a small inter-electrode capacity, which is rendered still further small by the fact that the electrodes connected to the same winding of the transformer are now in series instead of in parallel. With existing transducers, this would necessitate a large inductance to tune the capacity. The Q of the circuit would then be relatively large, and the bandwidth would be small. There are two approaches to a solution to this problem. One is to seek to increase the capacity per electrode by increasing the electrode width and decreasing the gap width, and to modify the response function by varying the ratio of finger width to meander-line width. The other is to design a much more sophisticated matching circuit than those in use hitherto, with a larger number of components. These methods might also be exploited

to increase the bandwidth of the transducer of Section IV. Work along these lines has not previously been carried out not because it is unreasonably difficult but because its necessity has not been apparent. The desirability of undertaking this work is now indicated by the results of Sections IV, V, and VI.

So far, we have been considering lumped-element matching circuits. These will be suitable for frequencies of the order of 100 MHz. At frequencies of 1000 MHz or higher, matching circuits will probably be realized in microstrip; this could be laid on the same substrate as the transducer in a monolithic process, once the design problems have been overcome.

VIII. CONCLUSION

The calculations of which results are given in this report indicate that transducers can be made to perform some quite sophisticated filtering operations. Before these can be realized, a great deal of work needs to be done on the questions of matching networks and of transducer performance in relation to the design of individual fingers. In this report I have tried to indicate the goals that may be attained if this work is carried out. But even with existing knowledge and techniques the transducer of Section IV should be realizable.

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13. ABSTRACT

By interweaving the fingers of an interdigital transducer with an earthed meander line and supplying different voltages, in different phases, to the fingers, a variety of interesting performances may be achieved. Two unidirectional transducers and a diplexing transducer are described as examples of the technique. In order to achieve the performances that theory indicates, a number of technical problems need to be overcome. These problems are pointed out and approaches to their solution are suggested.

14. KEY WORDS

interdigital transducers
elastic waves
microsound